Exam Seat No:_______________ Enrollment No:__________________

C.U.SHAH UNIVERSITY

WADHWAN CITY

University (Winter) Examination -2013

Course Name :B.Tech Sem-I **Subject Name: - Engineering Mathematics-I** **Marks :70**

Duration :- 3:00 Hours **Date : 16/12/2013**

Instructions:-

(1) Attempt all Questions of both sections in same answer book / Supplementary.
(2) Use of Programmable calculator & any other electronic instrument is prohibited.
(3) Instructions written on main answer Book are strictly to be obeyed.
(4) Draw neat diagrams & figures (If necessary) at right places.
(5) Assume suitable & Perfect data if needed.

SECTION I

Q-1 (a) Determine the region in z-plane represented by $Re(z) > 3$ 01

(b) Find polar form of complex number $z = -\sqrt{3} + i$ 02

(c) Evaluate $\lim_{x \to \pi/2} \frac{\log \sin x}{(\pi - 2x)^2}$ 02

(d) Define absolute and conditional convergence of series. 02

Q-2 (a) (i) Use definition of differentiation to find $f'(x)$ if $f(x) = \frac{x}{x-1}$ 02

(ii) Find the complex number z if $\arg(z+1) = \frac{\pi}{6}$ and $\arg(z-1) = \frac{2\pi}{3}$ 03

(b) (i) Test the convergence of series $\sum_{n=1}^{\infty} \frac{n^3+2}{2^n+2}$ 02

(ii) Prove that $\left(\frac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right)^n = \cos n\left(\frac{\pi}{2} - \theta\right) + i \sin n\left(\frac{\pi}{2} - \theta\right)$ 03

(c) Prove that n^{th} root of unity are in geometric progression. Also, show that their sum is zero. 04

OR

Q-2 (a) (i) Solve the equation $z^2 - (5+i)z + 8 + i = 0$ 02

(ii) Is $Arg(z_1 z_2) = Arg(z_1) + Arg(z_2)$? Justify. 03

(b) (i) Test the convergence of series $\left(\frac{1}{2}\right)^{1^2} + \left(\frac{2}{3}\right)^{2^2} + \left(\frac{3}{4}\right)^{3^2} + \cdots\cdots\cdots$ 02

(ii) Expand $\sin^5\theta \cos^3\theta$ in series of sine multiple of θ 03

(c) Evaluate the following limits: 04

(i) $\lim_{x \to 1} (1 - x^2)^{\frac{1}{\log(1-x)}}$ (ii) $\lim_{x \to 0} \left(\frac{1}{x^2} - \cot^2 x\right)$

Q-3 (a) (i) Evaluate $(1 + i\sqrt{3})^8 + (1 - i\sqrt{3})^8$ 03

(ii) Trace the curve $y^2(2a - x) = x^3$ 04

(b) (i) Find constants a, b, c so that $\lim_{x \to 0} \frac{x(a + b\cos x) - c\sin x}{x^5} = 1$ 03

(ii) Test the convergence of series $\frac{x}{2 \cdot 5} - \frac{x^2}{2^2 \cdot 10} + \frac{x^3}{2^3 \cdot 15} - \frac{x^4}{2^4 \cdot 20} + \cdots\cdots\cdots$ 04

16-B

OR

Q-3 (a) (i) Find the asymptotes of curve $x^2y^2 - x^2y - 5xy^2 + x + y + 5 = 0$ 03

(ii) Test the convergence of following series: 04

(1) $\sum \left[\sqrt{n^2+1} - 1\right]$ (2) $\frac{1}{1\cdot2} - \frac{1}{3\cdot4} + \frac{1}{5\cdot6} - \frac{1}{7.8} + \cdots\cdots\cdots$

(b) (i) Find and plot all roots of $\sqrt[3]{8i}$ 03

(ii) Find the radius of convergence and interval of convergence of series 04

$$-\frac{(x+2)}{1\cdot2} + \frac{(x+2)^2}{2\cdot2^2} - \frac{(x+2)^2}{3\cdot\ 2^3} + \frac{(x+2)^3}{4\cdot2^4} - \cdots\cdots\cdots$$

SECTION II

Q-4 (a) State Euler's Theorem on Homogeneous Function 01

(b) Find the Maclaurin's series of $\log(1-x)$ 02

(c) Find $\frac{\partial(u,v)}{\partial(x,y)}$ for $u = x^2 - y^2, v = 2xy$ 02

(d) If $u = e^{xyz}$, find $\frac{\partial^3 u}{\partial x\,\partial y\,\partial z}$ 02

Q-5 (a) (i) Prove that $\sqrt{(1+\sin x)} = 1 + \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{48} + \cdots\cdots\cdots$ 02

(ii) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ 03

(b) (i) Find the linearization of $f(x,y,z) = x^2 - xy + 3\sin z$ at the point $(2,1,0)$. 02

(ii) Use partial derivatives to find $\frac{du}{dx}$ if $u = \tan^{-1}\left(\frac{x}{y}\right)$, $x^2 + y^2 = a^2$ 03

(c) If $x = \frac{\cos\theta}{u}$, $y = \frac{\sin\theta}{u}$, evaluate $\left(\frac{\partial x}{\partial u}\right)_\theta \left(\frac{\partial u}{\partial x}\right)_y + \left(\frac{\partial y}{\partial u}\right)_\theta \left(\frac{\partial u}{\partial y}\right)_x$ 04

OR

Q-5 (a) (i) Find Taylor's series expansion of $f(x) = x^3 - 2x + 4$, $a = 2$ 02

(ii) If $w = xy + yz + xz$, $x = u + v$, $y = u - v$, $z = uv$,find $\frac{\partial w}{\partial u}$ & $\frac{\partial w}{\partial v}$ at $(u,v) = \left(\frac{1}{2}, 1\right)$ 03

(b) (i) Find the equation of tangent plane and normal line to the surface 02

$x^2 + 2y^2 + 3z^2 = 12$ at the point $(1,2,-1)$

(ii) Discuss the continuity of $f(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2} & ,(x,y) \neq (0,0) \\ 0 & ,(x,y) = (0,0) \end{cases}$ at $(0,0)$. 03

(c) If $u = \sin^{-1}\left(\frac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{x^{\frac{1}{5}}+y^{\frac{1}{5}}}\right)$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ and $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ 04

Q-6 (a) (i) If $V = r^m$, where $r^2 = x^2 + y^2 + z^2$ then prove that 03

$V_{xx} + V_{yy} + V_{zz} = m(m+1)r^{m-1}$

(ii) Expand $\tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x-1)$ and $(y-1)$ using Taylor's expansion. 04

	(b)	(i) Find percentage error in area of an ellipse when errors of 2% and 3% are made in measuring its major and minor axes respectively.	03
		(ii) A soldier placed at point (3,4) wants to shoot the fighter plane of an enemy which is flying along the curve $y = x^2 + 4$ when it is nearest to him. Find such distance by using Lagrange's multipliers method.	04

OR

Q-6	(a)	(i) Expand $\sin^{-1} x$ up-to the first four terms by Maclaurin's series.	03
		(ii) Show that $JJ' = 1$ for $x = e^v \sec u$, $y = e^v \tan u$	04
	(b)	(i) Find the extreme values of function $x^2y - xy^2 + 4xy - 4x^2 - 4y^2$	03
		(ii) If $f(xy^2, z - 2x) = 0$, show that $2x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 4x$	04

B***16****TECH****